

The Accelerating Universe – An Alternate Heuristic Thesis

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Abstract

Recent observations have revealed a perplexing property of the far reaches of the ancient universe. Rather than decelerating and presumably reversing direction and returning to its physical origin as is postulated for a closed universe, or decaying asymptotically to a final velocity as postulated for an open universe, the ancient mass at the far reaches of the universe has been observed to be actually accelerating away from its origin. Various theories have been proposed to explain this unexpected property of the universe. This work deviates substantially from those theories and posits that it is the universe mass itself which is the attractant attracting itself via conventional gravitational forces reaching around space-time. According to this thesis, this evidence of acceleration of this universe mass witnesses that the universe is closed.

1 THE ACCELERATING UNIVERSE

Recent observations of the cosmos suggest that the expansion at the far reaches of the universe is accelerating [1]. Contemporary cosmological theories cannot account for this phenomenon. In order to embrace this new evidence, various modifications to contemporary theories are being entertained including expanding the number of mathematical dimensions, introducing such parameters as dark energy, X-matter and quintessence, and rethinking the cosmological constant [2].

These new theories are generally all very complex approaches to reconciling the new evidence, and some of these themselves require the development of further theoretical concepts that, at this point, remain undefined.

Another approach to understanding this new and provocative evidence is to apply Ockham's Razor using reasonably-understood and generally-accepted cosmological precepts to explain the observational evidence.

2 THE UNIVERSE ATTRACTANT

It is a generally-accepted concept that due to the curvature of space-time, if one, with adequate means of transport and sufficient time, were to attempt to

travel to the edge of the universe, he would eventually return to his departure point [3]. This concept can also be applied to gravity itself without the need for development of any new theoretical basis. For example, if a mass were by some means suddenly created at some point in the universe, the gravitational effect due to this mass element would travel away from this mass in all directions in space-time at the speed of light toward the edge of the universe. By the argument above, this gravitational effect would eventually return to its point of origin to exert a gravitational force on the mass element from which it originally emanated.

Consider that at the instant that this element of mass was created, it was instantiated with a velocity in three-space. The effect of gravity will still move away from this mass element in all directions at the speed of light with respect to the origin point of this mass element since the speed of light is a universal constant.

When the effect of gravity reaches around space-time to the original origin point of this mass element in three-space, the mass element will have moved due to its nascent velocity. Therefore, when the component of the gravitational effect which traveled out in the same direction in three-space as the mass-velocity vector, the "forward effect," returns to the origin point, the mass element will have moved away from this position. Accordingly, the mass element

* The Author is an engineer, not a cosmologist. The author deeply apologizes should these ramblings be considered totally outrageous to the art of cosmology. Should that be the case, the author prays that the strongest emotion evoked by this work is that of boisterous laughter. At least a good end will still have been served.

will experience no effect of this forward component of its own gravitational effect as this component of the gravitational effect arrives at the origin point.

However, for the component of the gravitational effect which traveled out in the opposite direction as the mass-element velocity, the “aft effect,” it will encounter the mass element before reaching the origin point since the mass element has moved “toward” this component of its own gravity. When this component of gravity reaches the mass element, a net gravitational force will be exerted in the direction of the mass-element velocity. This force will impart an acceleration to the mass element according to simple Newtonian physics. To a remote observer observing this mass element after this aft component of its own gravity has reached the mass, it will appear that some unknown force is pulling, or conversely pushing, this mass element resulting in an acceleration.

The forward component of gravity which traveled in the direction of the initial mass-element velocity, when arriving at the mass-element origin point, sadly finds the mass missing. This gravitational effect will continue toward the moving mass element until it too eventually reaches the mass element. It will then also exert a force on the mass element restraining the acceleration.

However, since the effect of gravity is a square-law effect with distance, and since the physical path length of the forward gravitational effect around space-time to the mass element is longer than that of the aft gravitational effect, the aft gravitational effect will exert a greater force on the mass element than that of the forward effect.

Therefore, when both the forward and aft gravitational effects have fully entertained the mass element, there will remain a net forward gravitational force, and accordingly a net forward acceleration. The net effect of these two gravitational forces will result in a predictable, analytic velocity and acceleration profile of the mass element.

Such a spontaneous creation of mass instantiated with some nascent velocity is not simply theoretical. One such spontaneous creation of mass was the creation of the universe itself.

Consider that rather than some yet-to-be-discovered process at work at the outer reaches of the universe, it is instead the mass of the universe itself which is attracting and in turn accelerating the oldest mass of the universe. Rather than some unknown, unseen force such as dark energy, consider that the oldest mass of the universe has reached a point in space-time where the gravity due to the mass of the universe is attracting and accelerating this oldest

universe mass through “conventional” gravitational forces reaching around the space-time trajectory.

3 ORIGIN OF MASS AND GRAVITY

There was no actual point in time of creation of mass following the birth of the universe, but rather mass has always been present from the initial birth of the universe, and its gravitational effect was also present very early following the universal birth.

The contemporary views of the events of the birth of the universe generally hold that time, space, and all essence of the universe emerged at the instant of initial emergence of the birth singularity. The first of the fundamental forces to emerge was Gravitation. This was followed by the emergence Strong Nuclear Force and finally by the separation of the Weak Nuclear Force and Electromagnetic Force. It was only after some additional time interval that the elementary particles appeared. It is at this point that actual particles carrying mass appeared. Accordingly, actual physical mass appeared late in the growth of the universe in comparison to the emergence of, time, space and the four forces. Since these mass particles appeared late in the growth of the universe, the gravitational effect of their mass also occurred at that same late time.

However, at the instant of the birth of the universe, all of the energy of the universe would have resided in some state if conservation of energy is assumed to hold.¹ In the early universe at the time of the emergence of Gravitation, energy would have therefore been present. Since gravity is a consequence of mass, then by the mass-energy equivalence this energy would have an associated gravitational field. Accordingly, the gravitational effect of this energy emerged from the birth singularity intimately with space-time from initial birth of the universe. This reasoning concludes that the mass equivalent has always existed, e.g., from the initial birth of the universe.

As the universe expanded from its birth beyond the point in time where Gravitation emerged to the point in time were the Weak Nuclear Force and the Electromagnetic Force separated, to the best of present understanding the universe expanded at the speed of light as manifest in the state of the universe

¹ The nature of this energy state may not be known, but if conservation of energy is to hold, then all energy comprising the universe emerged at the birth of the universe, and no new energy was “created” as the universe inflated and expanded.

at that time.² Also, as is presently held, the gravitational effect similarly traveled at the speed of light. Therefore, as the universe progressed to this point, the gravitational effect of the early universe exerted a gravitational force on the universe itself. Accordingly, this early effect of Gravitation would have resulted in bending of space-time from the initial birth of the universe.

The photon is the seat of the Electromagnetic Force. Therefore, with the separation of the Electromagnetic Force, universal energy would have begun to coalesce into photons by conservation of energy, and photons eventually emerged.

A single photon possesses both energy and momentum, but has no physical mass. However, a pair of identical photons traveling in opposite directions and taken as a system does exhibit mass.³ If it is argued that as photons emerged, such photonic pairs also emerged, mass therefore also emerged. This early universe would exhibit nascent mass even though no mass particles would yet be in existence.

Since the photons in a photonic mass system are constrained to travel in opposing directions at the speed of light, and if the photonic system were instantiated with zero net velocity, the nascent photonic mass would appear as a stationary point mass at the point of the birth singularity, both in three-space and space-time, as space-time, energy, photons and the gravitational effect traveled out from the birth singularity.⁴ Specifically, at this point in the progression of the universe, there would be no individual mass particles moving from the birth singularity.

As space-time progressed from the birth singularity, the universe grew at the speed of light (the speed of light in the specific state of the universe at the time). Contemporary theories hold that the effect of gravity also moves at the speed of light.

Therefore, the gravitational effect, both as a consequence of the nascent universal energy and the photonic-system mass, continued to travel out in all directions in space-time at the speed of light. To this point in the universal progression, the seat of the

gravitational force was effectively a single point mass at the point in space-time of the birth singularity.

At some point late in the progression of the universe, elementary particles appeared. Again by conservation of the energy, these particles coalesced from the universal energy.⁵ When the elementary particles appeared, the universe had reached some non-zero dimension.

Unlike the nascent photons, each of these elementary particles individually exhibited mass, and as a consequence of this mass, each elementary particle at the instant of its emergence contributed a gravitational effect to the universe. This gravitational effect due to the mass of an elementary particle, at the instant the particle emerged, moved out from the particle in all directions in space-time at the speed of light. Since space-time was being bent by the gravitational effect of the effective mass of the nascent universe, and the speed of light is a universal constant independent of the velocity of any frame of reference, the gravitational effect of each elementary particle followed the curvature of space-time as this gravitational effect traveled away from its host particle.

Since the mass particles coalesced from photons, and since these photons exhibited momentum, by conservation of momentum, the mass particle too would exhibit momentum. Accordingly, the mass particle was instantiated with a velocity when it appeared.

It is this property of the effect of gravity, as a consequence of a created mass particle having a velocity at its instantiation, following a curved space-time trajectory which is the fundamental basis of the thesis of this work.

4 A CLOSED UNIVERSE AN ALTERNATIVE THESIS

One common three-dimensional model of the universe is that of an expanding sphere expanding in three-space from a single initial "Birth Singularity," or Big Bang. It is hypothesized that if the total universe mass is sufficient, the universe is closed and the expansion will reach some maximum limit and then due to gravity will collapse back upon itself in a Big Crunch – the antithesis of a Big Bang. In this model, the center of mass of the universe in three space remains fixed at the point of the original Birth Singularity. The Big Crunch will, in a "Death Singularity," return all of the universe mass, all space

² This assumes that all of the fundamental laws of physics were also "born" at the birth of the universe, and remained unchanging as the universe progressed in space-time. However, it has been postulated that space itself traveled faster than the speed of light.

³ This concept is well treated in the art, and therefore is not explored in this work.

⁴ As noted, any photons not paired with a companion to form a system would contribute no physical mass to the nascent universe.

⁵ It has been postulated that the elementary particles coalesced from the early photons.

and all time back to the same point in three-space as the Birth Singularity.

As an alternative thesis, consider a universe model where the center of mass of the universe is not defined in terms of three-space, but rather moves along a space-time trajectory. A simple two-dimensional Gadanken model of this concept is shown in Figure 1.⁶

From the initial Birth Singularity, the universe, gravity and space-time itself, expand out along a curved space-time trajectory. This trajectory is curved due to the mass of the universe bending space-time. In the model of Figure 1, as the universe begins to expand from the Birth Singularity, all universe mass is initially moving away from all other universe mass. Gravity, which is a consequence of the mass of the universe itself, extends in all directions in space-time including forward in space-time.

As the universe mass at the outer reaches of the universe, i.e., the “oldest” universe mass, moves along the space-time trajectory, the effect of gravity reaches forward along the space-time trajectory ahead of the oldest universe mass.⁷ At some point in space-time, the gravity which is a consequence of the mass of the universe reaches around the space-time trajectory such that a gravitational force is exerted on the oldest universe mass resulting in this oldest universe mass being pulled forward in space-time. At this point the older universe mass begins to accelerate along the forward space-time trajectory.

When the oldest universe mass reaches point λ in this Gadanken model, these gravitational forces along the forward space-time trajectory overcome the gravitational forces of the younger universe mass acting on the older universe mass.⁸ As universe mass passes point λ , it will begin to accelerate forward along the space-time trajectory due to these gravitational forces acting along the forward space-time trajectory. At some point in the age of the universe, all universe mass will have passed λ in

space-time. At that point, all universe mass will be moving toward all other universe mass in space-time, and all universe mass will be accelerating toward the Death Singularity.

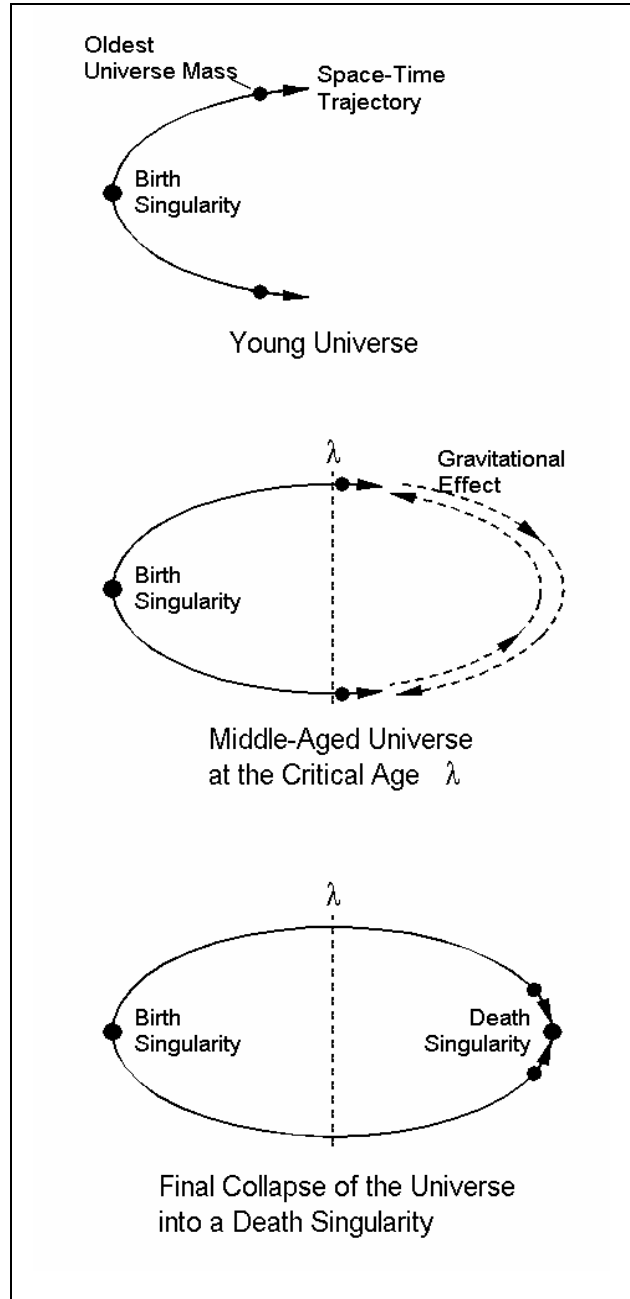


Figure 1. Two-Dimensional Universe Model

⁶ The drawings of Figure 1 are not renderings of a proposed surface of the universe but rather a timeline, and more specifically a space-timeline. Therefore, all of space, all mass and all time are constrained to be on this space-timeline.

⁷ This assumes that the mass velocity is less than the speed of light, the effect of gravity moves at the speed of light, and that the speed of light is a universal constant.

⁸ The symbol λ was chosen as a form of respect for Dr. Einstein – his Cosmological Constant is often represented with this symbol.

5 THREE-DIMENSIONAL UNIVERSE MODEL

This life of the universe may be more easily conceptually visualized in three-space. Consider the life progression of the universe modeled in Figure 2.⁹

This explanation is similar to that for Figure 1, but from a three-space perspective. Again, the trajectory is the space-time trajectory, or space-timeline. All of space, all mass and all time are constrained to this trajectory.

As soon as the universe is born at the Birth Singularity, the universe mass begins to move away from the Birth Singularity along the curved space-time trajectory. As noted, this trajectory is curved due to the mass of the universe bending space-time.

The universe follows this trajectory with the “oldest” universe mass eventually reaching the point λ in space-time. At this point λ , the effect of gravity has reached forward in space-time along the forward curved space-time trajectory to the point where it has just encountered the universe mass which has just reached point λ (this is the definition of the space-time point λ). At this point in space-time, gravity begins pulling the oldest universe mass forward in space-time away from the Birth Singularity.

To a real observer in a young universe, all universe mass will initially appear to be moving away from all other universe mass in an expanding universe, and the older universe mass will be observed to be decelerating. When the age of the universe reaches the point λ , the oldest mass at the farthest reaches of the universe from the Birth Singularity will appear to the observer to still be expanding, but the deceleration will be observed to stop, and the oldest mass will be observed to begin accelerating as it passes λ . The perception of the observer will be that there is some unseen force pulling (or conversely pushing) the outer reaches of the universe causing an acceleration in the expansion.

⁹ It should be clearly understood that the figures of the universe presented in this work show space-time trajectories, and are not intended to represent multi-dimensional surfaces. Accordingly, these figures depict a space-timeline equivalent to a timeline in more conventional understanding. Since these trajectories are space-time trajectories and not simply time trajectories, all of space, all mass and all time are constrained to reside on these trajectories.

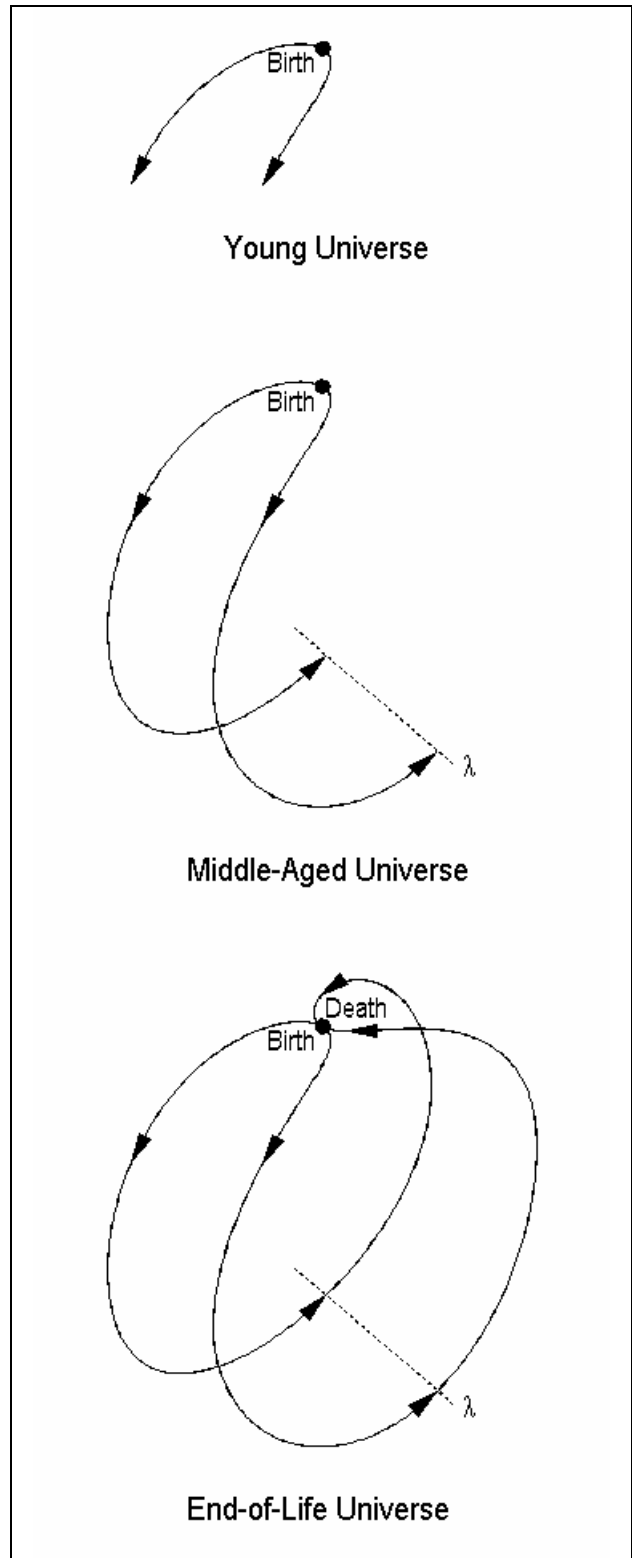


Figure 2. Three-Dimensional Universe Model

But in this Gedanken model, it is the gravity which is a consequence of the mass of the universe which is applying simple gravitational force to the oldest universe mass acting as an attractant pulling and accelerating the universe along the closed space-time trajectory toward a Death Singularity.

At some point in space-time, all universe mass will pass point λ . All universe mass will then be accelerating along the curved space-time trajectory toward all other universe mass due to the influence of gravity. But, to the real observer, all universe mass will be observed to be moving away from all other universe mass, and all mass will be observed to be accelerating away from all other mass.

To argue this reasoning, consider a real observer at some point along the space-time trajectory after all universe mass has passed point λ . This observer is constrained to observe only along the space-time trajectory. As the observer looks “forward” in space-time toward the “older” universe mass, that older mass is observed to be accelerating away from the observer. If the observer were to instead look “back” toward the “younger” universe mass, the observer is accelerating away from that younger universe mass. Since the observer has no fixed point of reference, the observer will observe the younger universe mass also accelerating away from the observer.

Accordingly, an observer at some point on the space-time trajectory after all universe mass has passed point λ cannot distinguish between “older” and “younger” universe mass since the observer has no fixed reference against which this distinction may be measured. Therefore, after all universe mass has passed point λ in space-time, an observer will observe all universe mass accelerating away from all other universe mass.

However, after all universe mass has passed point λ , all universe mass will actually be accelerating toward all other universe mass. Ultimately, all of space, all mass and all time will be drawn into focus at the Death Singularity. Since all of space-time and all mass originated in the Birth Singularity, and all of space-time and all mass is ultimately compressed into the Death Singularity, the Birth Singularity and the Death Singularity are the same entity.

Since the Birth and Death Singularities are the same entity, the final rendering of Figure 2 shows the Birth and Death Singularities as coincident in space-time. The resulting three-space figure shown in Figure 2 is reminiscent of the familiar “saddle shape” curved space-time which has been postulated for almost a century. The significant variation offered by this work is that two cusps of the saddle touch at the Birth/Death Singularity, or Death/Birth singularity

depending from what perspective in space-time this singularity is being considered.

6 A CLOSED UNIVERSE

Thus, according to this thesis, the evidence of acceleration of the universe mass at the farthest reaches of the universe witnesses that the universe is closed. However, it is not closed in the sense that it will collapse back upon itself. It is closed in the sense that it will continue forward in space-time and accelerate into itself coming to final focus in the Death/Birth Singularity.

7 CAN THIS THESIS BE TESTED?

It should be relatively straightforward to construct an elementary mathematical model with a limited number of particles to allow visualization of the thesis presented in this work. This would allow one to “see” the dynamical progression of the life of the mass of the universe as described in this work.

It may be feasible to loosely test this thesis using contemporary cosmological tools already perfected. Since evidence of an accelerating “expansion” of the universe has apparently been observed, some of the oldest universe mass has in our present time passed the critical point λ in the proposed Gedanken model. However, not all universe mass is observed to be accelerating away from the observer. Therefore, not all universe mass has yet passed the point λ in the model of this work.

Accordingly, within the universe there should be a distribution of velocities and accelerations of the universe mass as a function of the “age” of the universe mass. Younger mass will present with the highest velocities. The older the mass ranging from the Birth Singularity to point λ in space-time, the lower the velocity of that mass. Beyond point λ , the velocity of the mass should increase with age, and this mass beyond point λ should be observed to be accelerating away from the observer.

Therefore, there should be a distribution of velocity of the universe mass ranging from comparatively high-speed young mass, to the lowest velocity for older mass near the critical point λ , and then increasing velocities for the oldest mass beyond the critical point λ with that older mass observed to be accelerating away from the observer. Mass at the critical point λ will be at constant, minimum but nonzero velocity. Thus, there are three very distinct boundary conditions which are observable and can be objectively distinguished: decelerating mass, constant-velocity mass, and accelerating mass. These

three boundary conditions apparently have actually been observed.

If the laws of physics hold from the earliest “moments” just after the birth of the universe to the very final moments just before death, it is reasonable to postulate that there will be symmetry in the expansion and contraction of the universe about point λ . Therefore, if point λ can be found in the present universe where some universe mass has stopped decelerating and has just begun accelerating, and if a competent age can be determined for that mass at the critical point λ , the lifetime of the universe can be estimated as twice the time to the critical point λ in space-time.

8 CONCLUSIONS

The recent evidence of acceleration of portions of the universe mass has perplexed cosmology. The contemporary understanding of the cosmos cannot yet reconcile this new evidence. This work briefly reviews a theory which embraces this new evidence and applies Ockham’s Razor to explain that the acceleration of portions of the universe mass witnesses a closed universe. According to this thesis, it is the gravitational effect as a consequence of the mass of the universe itself which is attracting the oldest universe mass at the outer reaches of the universe resulting in an acceleration of this oldest mass toward a final Death Singularity. Further, space-time is a curved structure where two cusps are coincident in a singularity. This singularity is both the Birth Singularity of the universe and the Death Singularity of the universe.

Using contemporary cosmological tools and data, it may be possible to discover universe mass which has just arrived at a critical point λ in space-time where it is neither decelerating nor accelerating, and it may be possible to reasonably estimate the age of this mass. If such mass can be found, and its age reasonably estimated, and if the laws of physics hold from the first moments after the birth of the universe to last

moments before the death of the universe, it may be possible to estimate the lifetime of the cosmos.

This thesis proposes that the acceleration of the universe mass witnessed at the outer reaches of the universe is evidence that the universe is closed. However, it is not closed in the sense that the universe will collapse back upon itself, but rather it will be accelerated forward in space-time due to gravity reaching forward in space-time pulling the universe into focus in a final Death Singularity. Since all of space, all mass and all time are collected into the Death Singularity, the Death Singularity is indistinguishable from the Birth Singularity from which the universe emanated. The Death Singularity and Birth Singularity are the same entity. Accordingly, this thesis proposes that the universe shall return to its origin, and therefore the universe is closed.

At this point, science must likely defer to theology and philosophy to conjecture whether all space-time simply disappears, or whether a new universe is reborn from the Death Singularity. Is this Death Singularity simply the Birth Singularity of the next incarnation of the cosmos – the Cosmic Dance of Shiva as referenced by the late Dr. Sagan [4]? It would indeed be an interesting coincidence if such a computation of the lifetime of the cosmos were to be substantially equal to the Brahma Lifetime.

9 REFERENCES

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Author’s Note:

If the reader has comments (positive or negative) or suggestions concerning this work, the author would be eager to review any such comments. This is a living work. Admittedly, the concepts presented are rather difficult to envision. As the author receives suggestions for improving the presentation of the concepts reviewed, he will continue to modify this work to try to make the information more accessible.

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AUTHOR'S THESIS DEVELOPMENT NOTES

M. E. Gruchalla, April 8, 2008

I originally envisioned the concepts reviewed in this work in 1980 and 1981 following my reading *Cosmos* and viewing of the *Cosmos* series. At that time, I thought that if the cosmos was curved as postulated for many years, why then did not gravitational forces reach around the cosmos, around the space-time continuum, and pull the most ancient mass away from its origin, e.g., away from the Big Bang. If this were the case, then the most ancient mass of the universe should be accelerating away from its point of origin. Figure 2 of this work is substantially my original sketch from my initial conceptional developments. I subsequently unfolded the image of this figure to a type of flattened two-dimensional projection to use as a simplified tool to explain my thesis. Figure 1 of this work is substantially my original two-dimensional sketch.

I discussed my thesis with several individuals during the 1980's time frame. But, my concept was a preposterous thesis in the 1980's – and perhaps it is still so. There was no cosmological evidence of any type of acceleration whatsoever of ancient universe mass. However, Hubble's Law and the Hubble Constant had been an accepted concept since the late 1920's. Hubble's Law posits that the red shift of galaxies and all cosmological entities, e.g., the velocity relative to earth, is directly a function of the distance from the earth. – the farther away an object, the faster it is receding.

My argument is that since there is a proven distribution of velocities of the cosmological entities, physics as we understand it requires that there must at some time have been acceleration. And indeed this acceleration could still exist in the present time. However, the prevailing theories in the 1980's were more or less that all universe mass was instantiated with its inherent velocity at the instant of the Big Bang, and all universe mass has simply been moving out in space-time since the Big Bang.

It seemed to me, as well as others, that if this were the case, then the center of mass of the universe is fixed in three-space. Gravity as a consequence of the entire universe mass as soon as gravity emerged after the Big Bang should then have immediately begun exerting a restraining gravitational acceleration force on all universe mass causing all universe mass to immediately begin decelerating after the Big Bang. This was not supported by observational evidence.

In the 1980's, there was considerable debate as to whether the universe is closed where it will cease expanding and will be pulled back to the Big-Bang origin in a big crunch, or is open where all universe mass will reach a final terminal velocity and expand forever eventually dissipating into a gas of elementary particles. There was apparently even a Goldilocks Thesis where everything is just right for a stationary universe.

My thesis teaches that the universe is closed. But it is closed in a very unconventional sense. As reviewed in my work, according to my thesis, the universe will be drawn into focus at a Death Singularity, e.g., the Big Crunch. This Death Singularity will not be at the same physical point in three-space as the Big Bang Birth Singularity. However, it will be at the same point in space-time. Specifically since all of space, all mass and all time originated at the Birth Singularity, and all of space, all mass and all time are returned to the Death Singularity, the Birth and Death Singularities are the same entity. Therefore, the universe is closed where all of space, all mass and all time are returned to its origin.

I initially (late 1980's and early 1990's) attempted to contact several prominent physicists and cosmologists to ask if they would comment on my thesis. Admittedly, these individuals are continuously barraged by individuals thrusting arcane theories for review, and very likely theories much more plausible than mine, and these individuals understandably simply do not have the time to respond. But, I find this somewhat disappointing. I at least hoped for a resounding rejection of my thesis on the grounds that it was just too outrageous to be considered.

One of my peer reviewers of my initial works noted that I had not considered that mass actually existed in the form of its energy equivalent from the initial birth of the universe, e.g., universe mass has always been present from the initial emergence of the universe rather than being created at some late time after the Big Bang. In later works, I added the section on the origin of mass and gravity to embrace this concept and to reconcile this with my thesis.

This is a living work. I have discussed my concepts with many individuals over the years, and have had them review my various draft manuscripts from time to time. I have made many revisions to the original work to make the work more succinct and better focused in an attempt to clearly express my concept. As I discover more information and receive comments, I continue to revise this work.

Thanks,
Mike Gruchalla.

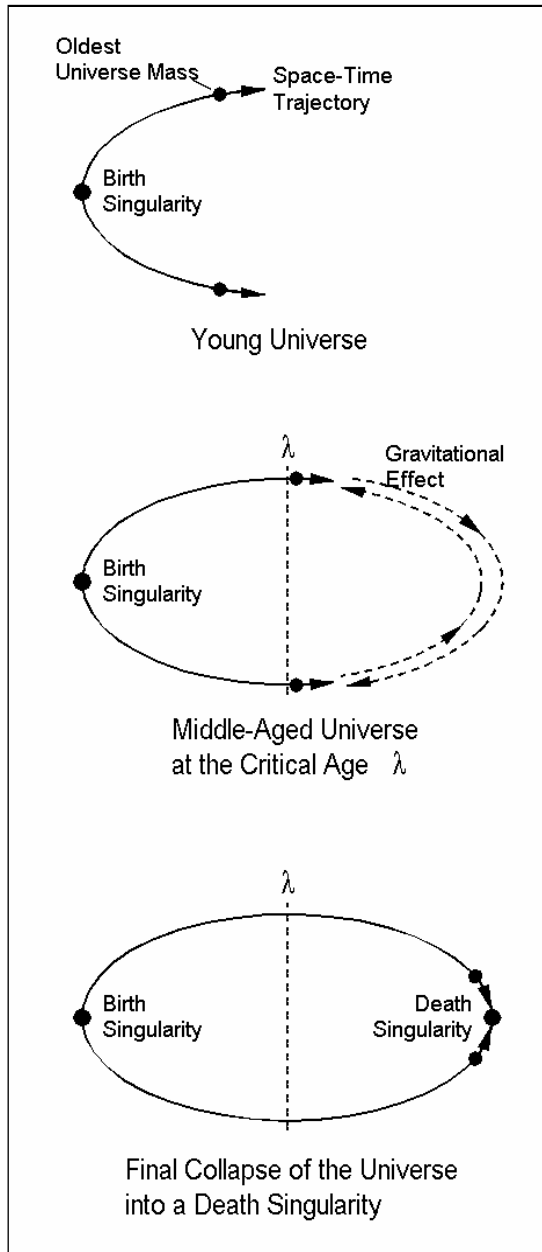


Figure 1. Two-Dimensional Universe Model

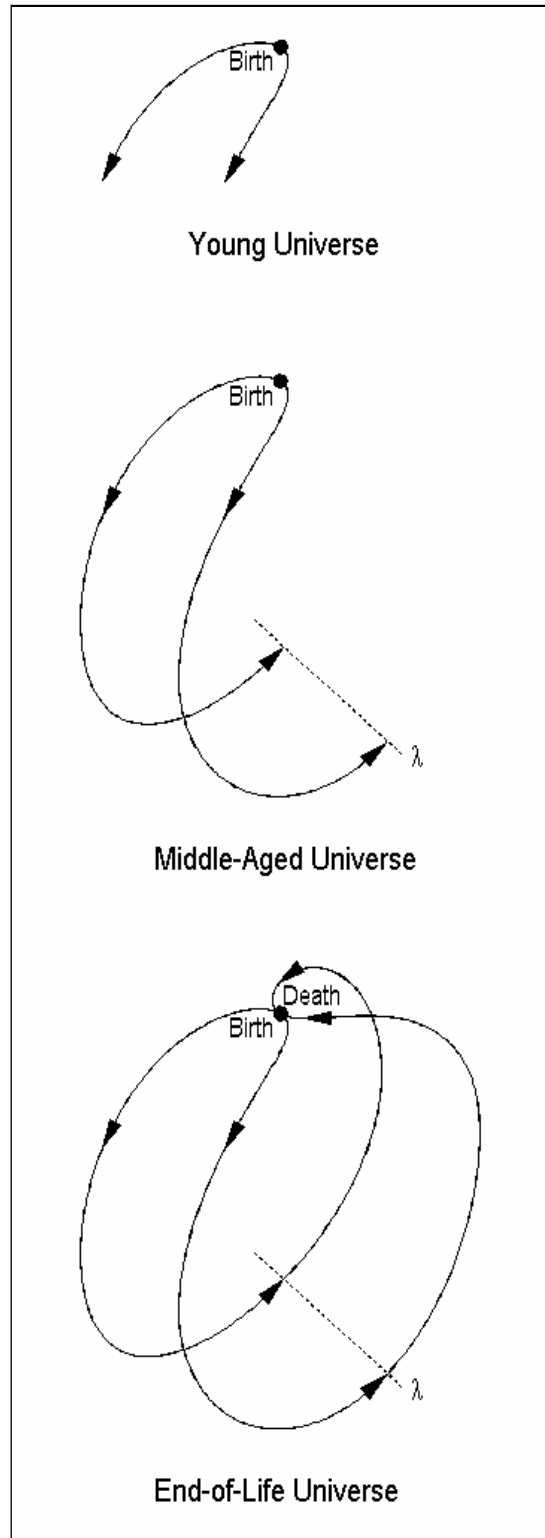


Figure 2. Three-Dimensional Universe Model